Numerical Analysis for Characterization of a Salty Water Meter

José Enrique Salinas Carrillo
Departamento de Ciencias Básicas
Instituto Tecnológico de Tehuacán
Bonfilio Arango Perdomo
Departamento de Mecatrónica
Instituto tecnológico de Tehuacán

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In this paper, It is Calculated and reported a part of the numerical characterization of the problem of a salty water meter. The salty water meter is a primary prototype which it is being studied in accordance with it´s technological applications proposed for the salt producers in Zapotitlan Salinas, a little village in the region of Tehuacan, in the state of Puebla, Mexico.

The searching of a formula for the area of two semicircle sections connected with a semi cylindrical section with a plate inside of this, It is an easy and clear problem that has already been solved by the geometricians. But from the point of view of the numerical methods, it results interesting the formula for the area through only numerical calculations.

Resumen

En este artículo se calcula y reporta parte de la caracterización de el problema de un medidor de salinidad del agua. El medidor de salinidad del agua es un prototipo primario el cual es estudiado de acuerdo a sus aplicaciones tecnologicas propuestas para los productores de sal de la villa de Sapotitlan Salinas situado en la región de Tehuacán, en el estado de Puebla, México.

La busqueda de una formula para el area de dos secciones semicirculares conectadad por un semi cilindro con una placa dentro de este arreglo, es un problema ya resuelto por los geometras, pero en este articulo es abordado desde el punto de los métodos numéricos y resulta interesante el análisis hecho para esta área, a través de únicamente cálculos numéricos.

Introduction

The numerical methods are appropriated for solve integrals, derivatives, linear and non linear problems, or differential equations. They have the characteristic of applying the programmed solutions to obtain the solution which could be difficult or impossible for obtain by another method. But not only are these cases solved by the programmers, but also the basic problems that had been already solved by another method. It is the case reported in this paper, the solution to the proposed problem to find the area of a semi cylindrical closed by the sides and with a planar plate inserted in the middle. It could result trivial from the point of view of the classical geometry. The solution is for the semi cylindrical surface $A_c = \pi rl$ and for the two semicircles $A_{ss} = \pi r^2$ and the planar plate $A_pp = ab$ where

$r = ratio, l = length\ of\ the\ cilinder,\ a = length\ of\ the\ planar\ plate\ ,$ $b = stregh\ of\ the\ planar\ plate$

Also to solve this problem by the numerical point of view, it would result useful for future calculations. We have in mind to solve the electromagnetic problem to determine the Electric Field, The Electric Potential and to obtain the equations for the capacity of the device formed by the experimental array involved in this primary approximation. Another possibility is to solve the equation to determine the resistance for the array working as a variable resistor.

Development

To suppose the geometry given by the figure 1 where the core of the cylindrical form is closed by the extremes and a planar plate is into the middle of the semi cylindrical recipient. In a previous paper (Salinas 2014) it had obtained the equations in terms of sets for the four surfaces supposing the references how are showed in the figure 2, these are the following sets:

$S_c = CilindricalSurface, f_b = Facebefore, f_p = FacePosterior, P_m = Mediumplate$

$S_c = (x, y, z) \big| 0 \leq x \leq l, z \leq 0, z^2 + y^2 = r^2, -r \leq y \leq r$

$f_b = (x, y, z) \big| x = 0, -r \leq y \leq r, -\sqrt{(r^2 - y^2)} \leq z \leq 0$

$f_p = (x, y, z) \big| x = l, -r \leq y \leq r, -\sqrt{(r^2 - y^2)} \leq z \leq 0$

$P_m = (x, y, z) \big| \frac{(l-a)}{2} \leq x \leq \frac{(l+a)}{2}, y = 0, -b \leq z \leq 0$

These sets are obtained supposing the origin of the coordinated axes as are showed in the figure 3.

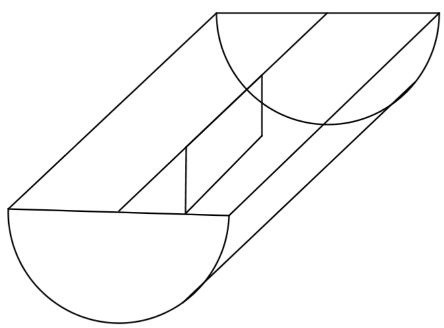
The goal is looking for the integral form for the surface of the device considered as the union of the sets

$S_c \bigcup f_b \bigcup f_p \bigcup P_m$.

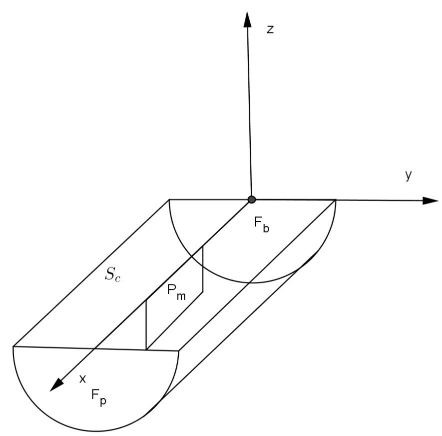
Let be a partition defined for S_c given for $x = [0, \frac{l}{n}, 2\frac{l}{n}, \ldots, (n-1)\frac{l}{n}, n\frac{l}{n}]$ and a partition for $z = [\frac{(-rn)}{n}, \frac{(-r(n-1))}{n}, \ldots, \frac{(-r2)}{n}, \frac{(-r)}{n}, 0]$ where x, y, z are measured on the cylindrical surface.

These two variables determine the values for y

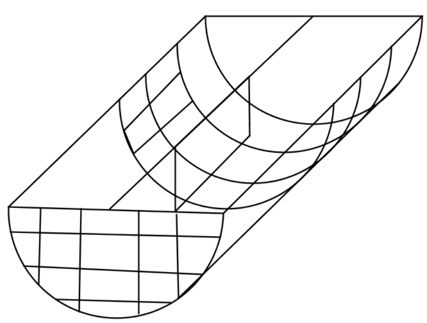
In the figure 2 we look at the differential of area between the points



(a) 1 Las piezas del transductor



(b) 2 Los ejes de referencia en el transductor



(c) 3 The Numerical Partitions

$(x_i, y_j, z_j), (x_i, y_{(j+1)}, z_{(j+1)}), (x_{(i+1)}, y_j, z_j), (x_{(i+1)}, y_{(j+1)}, z_{(j+1)}),$
and an approximation for ΔA is $\Delta A = \Delta x_i \sqrt{(\Delta y_j)^2 + (\Delta z_j)^2}$
Then the integral given by $A = \int_S dA = \sum_{i=1}^{n} \sum_{j=1}^{m} \Delta x_i \sqrt{(\Delta y_j)^2 + (\Delta z_j)^2}$

Where $y_j = \sqrt{r^2 - z_j^2}, y_{j+1} = \sqrt{r^2 - z_{j+1}^2}$

And $\Delta x_i = x_{i+1} - x_i, \Delta z_j = z_{j+1} - z_j, \Delta y_j = y_{j+1} - y_j = \sqrt{r^2 - {z_{j+1}}^2} - \sqrt{r^2 - z_j^2}$

Therefore an approximation for the integral is
$A = \sum_{i=1}^{n} \sum_{j=1}^{m} \Delta x_i \sqrt{(\Delta y_j)^2 + (\Delta z_j)^2}$ or
$A = \sum_{i=1}^{n} \sum_{j=1} (x_{i+1} - x_i) \sqrt{(z_{j+1} - z_j)^2 + \left(\sqrt{r^2 - {z_{j+1}}^2} - \sqrt{r^2 - z_j^2}\right)^2}$
Or in a simplified form expression
$A = \sum_{i=1}^{n} \sum_{j=1}^{m} (x_{i+1} - x_i) \sqrt{(z_{j+1} - z_j)^2 + 2r^2 - z_{j+1}^2 - z_j^2 - 2\sqrt{(r^2 - z_{j+1}^2)(r^2 - z_j^2)}}$

By other hand, the two areas of semicircular form can be partitioned by the following form: For $x = 0$, either it is the semicircular surface near the origin or for $x = l$, if it is the surface far front the origin, and for the $z_i, y_j$for both surfaces

$x_i = 0$,or $x_i = l$; $z_i = [\frac{-nr}{n}, \frac{-(n-1)r}{n}, ..., \frac{-(n-i)r}{n} \ldots, \frac{-1r}{n}, \frac{0r}{n}]$,
$y_j = [\frac{-nr}{n}, \frac{-(n-1)r}{2n}, \ldots, (\frac{-1.2r}{n+r}), \frac{0.2r}{n+r}]$ and the area could have been calculated using the approximation
$A = \sum_{i=0}^{n-1} \sum_{j=0}^{m-1} \Delta z_i \Delta y_j \chi(\|z_i^2 + y_j^2\| \leq r)$
Where$\chi(\|z_i^2 + y_j^2\| \leq r) = \begin{cases} 0 & if \quad \|z_i^2 + y_j^2\| \nleq r \\ 1 & if \quad \|z_i^2 + y_j^2\| \leq r \end{cases}$

Now, only is necessary to complete the surface with the plate part. The plate inside the body of the device has the next partitions: For $x \in [\frac{l-a}{2}, \frac{l+a}{2}]$ with $z \in [-b, 0]$

Where the differential of area it is given by $\Delta A = \Delta x_i \Delta z_j = (x_{i+1} - x_i)(z_{j+1} - z_j)$ And the area can be obtained adding the increments with
$A = \sum_{i=0}^{n-1} \sum_{j=0}^{m-1} (x_{i+1} - x_i)(z_{j+1} - z_j)$
But $x_i = \frac{l-a}{2} + \frac{ia}{n}$ con $i \in [0, n]$
And $z_j = -b + \frac{jb}{m}$ con $j \in [0, m]$
The area can be rewritten as follow
$A = \sum_{i=0}^{n-1} \sum_{j=0}^{m-1} \left(\left(\frac{l-a}{2} + \frac{(i+1)a}{n}\right) - \left(\frac{l-a}{2} + \frac{ia}{n}\right)\right) \left(\left(-b + \frac{(j+1)(b)}{m}\right) - \left(-b + \frac{jb}{m}\right)\right) = \sum_{i=0}^{n-1} \sum_{j=0}^{m-1} \left(\frac{a}{n}\right) \left(\frac{b}{m}\right)$

Numerical Results:

We ontain for a partition of $n = 10$,y $m = 10$ the values for $A_1,\ A_2,\ A_3$.reported into the table 1 for the true values we used $l = 21.2cm,\ r = 4cm,\ a = 4.5,\ b = 3.7cm$

and we applied the expresions $A_{t1} = \pi r l, A_{t2} = \pi r^2, A_{t3} = ab$

Table 1: Numerical Results of the aproximations with their errors

Set	n	m	Area cm^2	True Area cm^2	Error cm^2
A_1	10	10	2*132.808019	266.407057	0.791019837
A_2	10	10	22.08	25.13274123	3.052741229
A_3	10	10	16.65	16.65	3.90799E-14

and the values of the Errors are calculated by the formulas $E_1 = |A_1 - A_{t1}|$, $E_2 = |A_2 - A_{t2}|$, $E_3 = |A_3 - A_{t3}|$

Conclusions:

We have three expressions for the four surfaces A_1 is for the cylindrical surface

$A_1 = \sum_{i=1}^{n} \sum_{j=1}^{m} (x_{i+1} - x_i) \sqrt{(y_{j+1} - y_j)^2 + 2r^2 + y_{j+1}^2 + y_j^2 - 2\sqrt{(r^2 - y_{j+1}^2)(r^2 - y_j^2)}}$

$A_2$for the semicircles at $x = 0$ and $x = l$

$A_2 = \sum_{i=0}^{n-1} \sum_{j=0}^{m-1} \Delta z_i \Delta y_j \chi(\|z_i^2 + y_j^2\| \leq r)$

And $A_3$for the plate inside the body of the meter

$A_3 = \sum_{i=0}^{n-1} \sum_{j=0}^{m-1} (x_{i+1} - x_i)(z_{j+1} - z_j) = \sum_{i=0}^{n-1} \sum_{j=0}^{m-1} \left(\frac{a}{n}\right)\left(\frac{b}{m}\right)$

The numerical calculations for $n = 10$, $m = 10$ give $A_1 = 2{*}132.808019cm^2, A_2 = 22.08cm^2, A_3 = 16.65cm^2$

and the true values are $A_{t1} = 266.407057cm^2, A_{t2} = 25.1327412cm^2, A_{t3} = 16.65cm^2$ The errors are of $ErrorA_1 = 0.79101984cm^2, ErrorA_2 = 3.05274123cm^2$ and$ErrorA_3 = 3.90799E - 14cm^2$.

The true expresions are for the semicilinder$A_c = \pi rl$, for the two semicircles $A_{ss} = \pi r^2$, and for the planar plate $A_{pp} = ab$.

The greatest error is obtained in the semicircular sections, it could be because in the aproximation half of the elimined points add the half of area o the Total Area.

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References

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