The Quasy Rationals

Instituto Tecnológico de Tehuacan

December 16, 2013

Keywords: Axioms of numerical systems, Rationals, Processes

Abstract:

In this paper, it is proposed a series of systems denoted by $\mathbb{Q}A_n$ which had been reported previosly, but had not been analized in exaustive form. For the presentation of the given systems, firstly are reviewed the properties satisfied by the Naturals, The Integers, and The Rationals. It have done the observations of the necesity of the completedness of more complex systems by some requeriments not satisfied by the previous system.

Finally, it is analysed only one operation of the $\mathbb{Q}A_n$system. And by the similarity with the Rationals but some differences with this, It is named the Quasy Rationals System.

Resumen:

En el presente artículo, son propuestos una serie de sistemas denotados por $\mathbb{Q}A_n$ los cuales fueron reportados anteriormente, pero los cuales no han sido analizados exhaustivamente. En la presentación de dichos sistemas, primeramente son revisadas las propiedades que satisfacen los numeros Naturales, los Enteros y los Racionales. Haciendo incapie en la necesidad de completar los sistemas a sistemas más complejos debido a los requerimientos no satisfechos por los sistemas previos.

Finalmente es analizada solamente una operación de los sistemas $\mathbb{Q}A_n$, la operación denominada complemento y l-complemento. Por la similitud con el sistema de los racionales pero las diferencias que se presentan se propone el nombre de Sistema de Quasi-Racionales

Introduction

Usually, The real number system is considered composing by several subsystems included each one in another one. The sequence of includence between the subsystems is $\mathbb{N} \subset \mathbb{Z} \subset \mathbb{Q} \subset \mathbb{R}$

An approach to the description of these subsystems is followed by the construcction of the naturals applying the succesor function and defining the addition and multiplication by inductively form.

With this approach it is posible to show some basical properties, like the clousure, the commutative and asociative properties of the addition and multiplication into the $(\mathbb{N}, +, *)$ system, including the distributive law.

But, with the naturals, we can define the substration, elsewhere with this definition, the substration is not closed, because if $n - m \in \mathbb{N}$ then $m - n \notin \mathbb{N}$.

This is a reason for to extend the system of naturals to another system which would be closed with a posible equivalence between the $-$ operation defined as $n - m \doteq n + (-m)$ where $-n$ is the inverse additive of n.

It is clear that $n - m \in \mathbb{Z}$ and $m - n \in \mathbb{Z}$.

This result extend the definition given back to$(\mathbb{Z}, +, \cdot)$, other set which it is closed with the inverse additive definition and the neutro additive which is not into the Naturals because if $n - m \in \mathbb{N}$ then $m - n \notin \mathbb{N}$.

Something similar occurs when we can complete the $(\mathbb{Z}, +, *)$ system although if $n/m \in \mathbb{Z}$ then $m/n \notin \mathbb{Z}$.

This incomplete result induces to extend the integers system to another system named the Rational system. Whem the rational system is built we can show the result that, this system heredates the same properties than the $\mathbb{Z}$ has but aftermore it has the following properties.

$\forall \frac{p}{q} \in \mathbb{Q} \exists \left(\frac{p}{q}\right)^{-1} = \frac{q}{p} \in \mathbb{Q} \pitchfork \frac{p}{q} \cdot \frac{q}{p} = 1$

It is the existence of the inverse multiplicative and the aseveration that

$\frac{p}{q} \in \mathbb{Q} \rightarrow \frac{q}{p} \in \mathbb{Q}$ with $p \neq 0, q \neq 0$,where $p, q \in \mathbb{Z}$

But this paper pretends to present a new system of numbers which doesn't have all the properties of the $\mathbb{Q}$ system.

We can sugest that the $(\mathbb{Q}, +, \cdot)$ system consist of the set

$\mathbb{Q} = \left\{\frac{p}{q} | p, q \in \mathbb{Z}, q \neq 0\right\} \bigcup \{0\}$

with the properties

1. Closure property $\frac{a}{b} + \frac{c}{d} \in \mathbb{Q}$ y $\frac{a}{b} \cdot \frac{c}{d} \in \mathbb{Q}$
2. Conmutative property $\frac{a}{b} + \frac{c}{d} = \frac{c}{d} + \frac{a}{b}$ y $\frac{a}{b} \cdot \frac{c}{d} = \frac{c}{d} \cdot \frac{a}{b}$
3. Identique additive and multiplicative elements $\exists 0 \in \mathbb{Q} \pitchfork \frac{a}{b} + 0 = \frac{a}{b}$ y $\exists 1 \in \mathbb{Q} \pitchfork \frac{a}{b} \cdot 1 = \frac{a}{b}$
4. Inverse additive and inverse multiplicative $\forall \frac{a}{b} \in \mathbb{Q} \exists -\frac{a}{b} \in \mathbb{Q} \pitchfork \frac{a}{b} + (-\frac{a}{b}) = 0$ y $\forall \frac{a}{b} \in \mathbb{Q} \setminus \{0\} \exists (\frac{a}{b})^{-1} = \frac{b}{a} \in \mathbb{Q} \pitchfork \frac{a}{b} \cdot (\frac{a}{b})^{-1} = 1$
5. Asociative adding and multiplication $\forall \frac{a}{b}, \frac{c}{d}, \frac{e}{f} \in \mathbb{Q} \frac{a}{b} + (\frac{c}{d} + \frac{e}{f}) = (\frac{a}{b} + \frac{c}{d}) + \frac{e}{f} \wedge \frac{a}{b} \cdot (\frac{c}{d} \cdot \frac{e}{f}) = (\frac{a}{b} \cdot \frac{c}{d}) \cdot \frac{e}{f}$
6. Distributive property $\forall \frac{a}{b}, \frac{c}{d}, \frac{e}{f} \in \mathbb{Q} \frac{a}{b} \cdot (\frac{c}{d} + \frac{e}{f}) = (\frac{a}{b} \cdot \frac{c}{d}) + (\frac{a}{b} \cdot \frac{e}{f})$

But there exist only one option of set which can be as pairs of factors?. Are these properties everytime satisfied?. Or there exists another posibilities for the $\mathbb{Q}$ set?.

Well these questions were researched by E. Carrera F., I. Carrera M., D. López, A.Isaias K. M., M. Vázquez H. (1), G. Vázquez B., C. Mora R. (2) E. Jasso R., L. Montiel R.(3), F. Serrano L., D. Anaya G., N. Carrillo C., J.C. Guzmán C., J. C. Saavedra E.(4), E. Navarrete C., J. Aguilar I., G. J. Cruz M., M. A. Torres M., J. Velasco C.(5), and T. A. Castro P., I. Amil C., J. Tecua H., L. A. Valerio V.(6)

They reported the application of the scheme of the rational numbers to secuencial proccesses of 6, 7, 8 and 9 steps and they suggest that the axioms are usefull and they could be applied in the daylly activities but they found that not all the operations could have sence.

Development:

Although these reports were simple and they haven't forthermore transcendence in that time, mayb*e by the unknowing of the theme of the readers. I noted that these results sould be researched and analize*d w*ith more detail.*

For that reason now I present some new advances in this context.

First we suppose the series of the following systems:

Let $A_n = \{1, 2, ..., n\}$ we define $\mathbb{Q}A_n = \left\{\frac{p}{n} | p \in \mathbb{N} \wedge n \neq 0\right\}$where if $p = kn + r$ with $0 \leq r < n$ then the fraction $\frac{p}{n}$ can be represented as $k\frac{r}{n}$ a composed fraction with k a complete process and the fraction $\frac{r}{n}$ which indicates that the final process had been not completed.

But $\mathbb{Q}A_n$ depends of the n, therefore there exist diferent sets.

The set $\mathbb{Q}A_6$is studied by E. Carrera F. et all., in the process of lents elaboration and by T.A. Castro P., et all. when they analized the bottle elaboration proccess.

The set $\mathbb{Q}A_7$is considered by G.Vázques et. all., it is applied to the elaboration of a pen.

The set $\mathbb{Q}A_8$ is analized by E. Jasso R. et all., how help in the manufacturing of automoviles.

The set $\mathbb{Q}A_9$ is applied by F. Serrano L., et all. in the proccess of elaboration of lightening anounces, and it is analized by E. Navarrete C. et all, in the production of sugar.

All of them considered only sequencial processes which need the first steps have been completed before the next one.

One first operation between the $\mathbb{Q}A_n$elements is the named complement operation.

Suppose that you have a $\frac{p}{n} \in \mathbb{Q}A_n$ where $\frac{p}{n} = \frac{kn+p_1}{n}$ with $0 \leq p_1 < n$ then the complement operation is defined as $\left\{\frac{p}{n}\right\}^{'}$or by $\left\{\frac{p}{n}\right\}^{'k}$ by the following conditions.

$\frac{p}{n} + \left\{\frac{p}{n}\right\}^{'} = \frac{kn+p_1}{n} + \left\{\frac{p_1}{n}\right\}^{'} = \frac{kn}{n} + \frac{p_1+\{p_1\}^{'}}{n} = \frac{kn}{n} + \frac{n}{n} = \frac{kn+n}{n} = \frac{(k+1)n}{n}$

and $\frac{p}{n} + \left\{\frac{p}{n}\right\}^{'l} = \frac{kn+p_1}{n} + \left\{\frac{p_1}{n}\right\}^{'l} = \frac{kn}{n} + \frac{p_1+\{p_1\}^{'l}}{n} = \frac{kn}{n} + \frac{ln}{n} = \frac{kn+ln}{n} = \frac{(k+l)n}{n}$

how it is possible to observe with the operation complement simple $\{\}^{'}$a unit proccess is completed, otherwise $\{\}^{'l}$ other l-proccesses will be completed.

By otherwise the operation + could be insufficient for explain this kind of operatión. And morever if we consider that in the examples reported by the authors mentioned It haven't mentioned processes which could be executed in paralel manner.

Could result important to analized and to give the notation for this kind of mathematical entes, this will be executed in next papers.

Conclusions:

It is presented a soft analisis of the axioms which would be satisfied by the systems Naturals, Integers, and Rationals, this analisis is the basis for the presentatión of the $\mathbb{Q}A_n$ systems, a series of diferent notations subsystems. With the $\mathbb{Q}A_n$we introduce the operation $\{\}^{'}$and the extended operation $\{\}^{'l}$, whose are named the complement operation and the l-complement operation, with the first operation we are securing that the element $\frac{p}{n}$ is completed to the next integer where if $p = nk + r$, with $1 \leq r < n$ then $\frac{p}{q} + \left\{\frac{p}{q}\right\}^{'} = \frac{nk+n}{p}$

Notation:

$\exists$ is the quantifier exist.\left|\right.
$\forall$ for all
$\mathbb{Q}$ The Rationals Set
$\mathbb{Q}A_n$ The n-Quasy Rationals Set
$\in$ is element of
$\notin$ is not an element of
$\{\}^{'}$ complement operation
$\{\}^{'l}$ l-complement operation
$\wedge$ logical and operation
$\vee$ logical or operation

$\Cap$which one
$\{ \mid \}$ The set of —- which satisfies
$\mathbb{Q} \setminus \{0\}$The Rationals except the Zero
$\subset$Subset of
$\cup$ Union of sets
$(\mathbb{Q}, +, \cdot)$ The set of the Rationals with the add and the multiplication operations, named the Rational System.
$(\mathbb{N}, +, *)$ The set of Naturals with the add and the multiplication operations, named the Natural System.
$(\mathbb{Z}, +, *)$ The set of Integers with the add and the multiplication operations, named the Integers System.

Acknowledgements:

I want to thank to E. Carrera F., I. Carrera M., D. López, A.Isaias K. M., M. Vázquez H., G. Vázquez B., C. Mora R., E. Jasso R., L. Montiel R., F. Serrano L., D. Anaya G., N. Carrillo C., J.C. Guzmán C., J. C. Saavedra E., E. Navarrete C., J. Aguilar I., G. J. Cruz M., M. A. Torres M., J. Velasco C., and T. A. Castro P., I. Amil C., J. Tecua H., L. A. Valerio V. by the the previus analisis reported by then in the Revista de Ciencias Básicas, I.T. Tehuacán, vol. 1, Num. 1 sem. 1, año 1.

References:

E. Carrera. F., I. Carrera M., D. López L., A. Isaías K. M. and M. Vázquez H., Los racionales en el proceso de fabricación de lentes, Revista de Ciencias Básicas I.T.Tehuacán. Vol.1, Num. 1, Sem. 1, Año 1. pp4-7.

G. Vázquez B. and C. Mora R. , Matemáticas en el armado de un Lapicero., Rev. de Ciencias Básicas I.T. Tehuacán,Vol.1, Num. 1, Sem. 1, Año 1. pp8-9.

E. Jasso R. and L. Montiel R., Los racionales como auxilio en la manufactura de automoviles., Rev. de Ciencias Básicas I.T. Tehuacán,Vol.1, Num. 1, Sem. 1, Año 1. pp10-13.

F. Serrano L., D. Anaya G., N. Carrillo C. J., C. Gúzman C. J. and C. Saavedra E. Proceso de Fabricación de Anuncios Luminosos y los Racionales, Rev. de Ciencias Básicas I.T. Tehuacán,Vol.1, Num. 1, Sem. 1, Año 1. pp13-16.

E. Navarrete C., J. Aguilar I., G. J. Cruz M. and M. A. Torres M. and J. Velasco C., Producción del Azúcar en relación a los racionales, Rev. de Ciencias Básicas I.T. Tehuacán,Vol.1, Num. 1, Sem. 1, Año 1. pp17-20.

T. A. Castro P, I. Amil C., J. Tecua H. and L. A. Valerio V., Los números racionales en base al procesamiento de la Botella, Rev. de Ciencias Básicas I.T. Tehuacán,Vol.1, Num. 1, Sem. 1, Año 1. pp21-25.